Exercise 1. (*Longitudinal model*)
Consider the longitudinal model $\dot{a}_{\text{long}}(t) = \frac{-1}{T_a} a_{\text{long}}(t) + \frac{1}{T_a} a_{\text{long}_{\text{in}}}(t)$.

a) Discuss / apply the six modeling steps you know from the lecture on this model [Lunze, Regelungstechnik 1, pp 40-41], e.g.
   1. What is your modeling goal?
   2. Draw the block diagram. What is the output? What is the input?
   3. Find the state-space representation (which order is the system?).

b) Program the model in MATLAB (with `tf` and the Laplace-transform or by implementing the ordinary differential equations (ODEs) and simulating with `ode45`) / Simulink (transfer block diagram).

c) Simulate the model with different inputs and parameters (e.g. T_a). Observe and analyze the output.

Exercise 2. (*Point-mass model*)
Consider the point-mass model $\ddot{s}_x = a_x, \quad \ddot{s}_y = a_y, \quad \sqrt{a_x^2 + a_y^2} \leq a_{\max}$.

a) Discuss / apply the six modeling steps you know from the lecture on this model [Lunze, Regelungstechnik 1, pp 40-41], e.g.
   1. What is your modeling goal?
   2. Draw the block diagram. What are the outputs? What are the inputs?
   3. Find the state-space representation (which order is the system?).

b) Program the model in MATLAB (with `tf` and the Laplace-transform or by implementing the ODEs and simulating with `ode45`) / Simulink (transfer block diagram).

c) Simulate the model with different inputs. Observe and analyze the outputs.

d) Warn the user when the constraint on $a_{\max}$ is violated.

Exercise 3. (*Kinematic bicycle model*)
Consider the kinematic bicycle model

$$
\begin{aligned}
\dot{s}_x(t) &= v(t)\cos(\psi(t) + \beta(t)) \\
\dot{s}_y(t) &= v(t)\sin(\psi(t) + \beta(t)) \\
\dot{\psi}(t) &= \frac{1}{\ell_{wb}} v(t) \tan\delta(t) \cos\beta(t) \\
\dot{v}(t) &= \text{this exercise} \\
\dot{\delta}(t) &= \text{this exercise} \\
\beta(t) &= \tan^{-1}\left(\frac{\ell_r}{\ell_{wb}} \tan\delta(t)\right) \quad .
\end{aligned}
$$

a) Discuss / apply the six modeling steps you know from the lecture on this model [Lunze, Regelungstechnik 1, pp 40-41], e.g.

   1. What is your modeling goal?
   2. Draw the block diagram. What are the outputs? What are the inputs?
   3. Choose a behavior for modeling the change in speed v and steering angle δ.
   4. What are the model's states, parameters and inputs?

b) Program the model in MATLAB (implement the ODEs and simulate with `ode45`) / Simulink (transfer block diagram).

c) Simulate the model with different inputs. Observe and analyze the outputs.

d) How does the side slip angle β relate to the steering angle δ? `plot` the relationship in MATLAB.

e) Linearize the ODEs with a first-order Taylor series. Determine where this linearization is sensible.

f) Extend your model with constraints on acceleration and steering angle.

g) This model incorporates the speed v at the vehicle center with distance ℓ_r to the rear axle. Express the speed v_ℓ at any point along the longitudinal vehicle axis depending on the speed at the rear axle. How does v_ℓ relate to the steering angle δ and the distance to the rear axle along the longitudinal vehicle axis ℓ_r?

Exercise 4. (*Vehicle model comparison*)
The repo https://gitlab.lrz.de/tum-cps/commonroad-vehicle-models/tree/master/MATLAB provides an implementation of several vehicle models.

a) Add the point-mass model you implemented.

b) Adjust the kinematic single-track model to incorporate the side-slip angle β.

c) Compare the models. Use `testVehicle.m` as an orientation.